Code No: 19SHT102

R19

I B. Tech I Semester Regular Examinations, Jan - 2020

MATHEMATICS – I

(Common to ALL Branches)

Time: 3 hours | Max. Marks: 60

Note: Answer **ONE** question from each unit (**5 × 12 = 60 Marks)**

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UNIT - I

1. a) Define linear differential equation of first order. (6M)
   Solve $(1 + y^2)\, dx = (\tan^{-1} y - x)dy$.
   b) Solve $(1 + xy^2)\, \frac{dy}{dx} = 1$. (6M)

(OR)

2. a) Solve $(x^2\text{-}ay)dx=(ax\text{-}y^2)dy$. (6M)
   b) Solve $y(2xy+e^x)dx=e^x dy$. (6M)

UNIT – II

3. a) Define complementary function. (6M)
   Solve $(i)\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0;\ (ii)\frac{d^3y}{dx^3} + y = 0$.
   b) Solve $(D^3+4D)y=\sin 2x$. (6M)

(OR)

4. a) Solve $\frac{d^2y}{dx^2} - 4y = x\sinh x$. (6M)
   b) Solve $\frac{d^2y}{dx^2} + a^2y = \sec ax$. (6M)

UNIT – III

5. a) State Cauchy's mean value theorem and verify for (6M)
   f(x) = sinx and g(x) = cosx in [a,b].
   b) Using Taylor's theorem prove that $x - \frac{x^3}{6} < \sin x < x - \frac{x^3}{6} + \frac{x^5}{120}$ *for* $x > 0$. (6M)

(OR)

6. a) Expand $e^{\sin x}$ by Maclaurins series upto the term containing x^4. (6M)
   b) If $x > 0$, then prove that $x > \log(1+x) > x - \frac{x^2}{2}$. (6M)
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7. a) If $u = f(r)$ *and* $x = r\cos\theta, y = r\sin\theta$, prove that (6M)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f^{11}(r) + \frac{1}{r} f^{1}(r).$$

b) If $u = xyz, v = x^2 + y^2 + z^2, w = x + y + z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. (6M)

(OR)

8. a) Expand $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ up to 3rd degree terms. (6M)

b) Examine the maxima and minima for $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. (6M)

UNIT –V

9. a) Find the area of a plane in the form of a quadrant of the ellipse (6M)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

b) Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2 + y^2 + z^2)\,dxdydz.$ (6M)

(OR)

10. a) Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dxdy$ by changing to polar coordinates. (6M)

b) Find the volume of a sphere $x^2 + y^2 + z^2 = a^2$ by using triple integration. (6M)

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